Inference at $*$ 1 0
of proof for Lemma minus_imax:

1. $a : \mathbb{Z}$
2. $b : \mathbb{Z}$
$\vdash$ (-if $a \leq$z b then b else a fi) = if ($-a$) $\leq$z -b then -a else -b fi
by PERMUTE{1:n,
2:n,
3:n,
4:n,
5:n,
6:n,
7:n,
8:n,
9:n,
10:n,
11:n,
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36:n,
34:n,
33:n,
37:n,
38:n,
39:n,
40:n,
41:n,
42:n,
43:n,
44:n,
45:n,
46:n,
47:n,
48:n,
46:n,
49:n,
47:n,
50:n,
51:n,
52:n,
53:n,
54:n,
52:n,
51:n,
55:n}

1: . . . . .wf. . . . . NILNIL

$\vdash a \leq_{\mathrm{z}} b \in \mathbb{B}$
2: . . . . .wf. . . . . NILNIL

$\vdash \mathbb{B} \in \mathrm{Type}$
3: . . . . .wf. . . . . NILNIL

3. $a \leq_{\mathrm{z}} b = \mathrm{tt}$
$\vdash (a \leq_{\mathrm{z}} b = \mathrm{tt}) \in \mathbb{P}_1$
4: . . . . .wf. . . . . NILNIL

3. $a \leq_{\mathrm{z}} b = \mathrm{tt}$
$\vdash (\uparrow a \leq_{\mathrm{z}} b) \in \mathbb{P}_1$
5: . . . . .wf. . . . . NILNIL

3. $a \leq_{\mathrm{z}} b = \mathrm{tt}$
$\vdash (a \leq b) \in \mathbb{P}_1$
6: . . . . .wf. . . . . NILNIL

3. $a \leq_{\mathrm{z}} b = \mathrm{tt}$
$\vdash a \leq_{\mathrm{z}} b \in \mathbb{B}$
7: . . . . .wf. . . . . NILNIL

3. $a \leq_{\mathrm{z}} b = \mathrm{tt}$
$\vdash a \in \mathbb{Z}$
8: . . . . .wf. . . . . NILNIL

3. $a \leq_{\mathrm{z}} b = \mathrm{tt}$
$\vdash b \in \mathbb{Z}$
9: . . . . .wf. . . . . NILNIL

3. $a \leq b$
$\vdash (\text{-}a) \leq_{\mathrm{z}} \text{-}b \in \mathbb{B}$
10: . . . . .wf. . . . . NILNIL

3. $a \leq b$
$\vdash \mathbb{B} \in \mathrm{Type}$
11: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash ((\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}) \in \mathbb{P}_1$
12: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash (\uparrow(\text{-}a) \leq_{\mathrm{z}} \text{-}b) \in \mathbb{P}_1$
13: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash ((\text{-}a) \leq (\text{-}b)) \in \mathbb{P}_1$
14: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash (\text{-}a) \leq_{\mathrm{z}} \text{-}b \in \mathbb{B}$
15: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$

$\vdash$ (-a) $\in \mathbb{Z}$
16: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = tt
$\vdash$ (-b) $\in \mathbb{Z}$
17: . . . . .truecase. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$ (-b)
$\vdash$ (-b) = (-a)
18: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff
$\vdash$ ((-a) $\leq$z -b = ff) $\in \mathbb{P}_1$
19: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff
$\vdash$ ($\uparrow$(-b) <z (-a)) $\in \mathbb{P}_1$
20: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff
$\vdash$ ((-b) < (-a)) $\in \mathbb{P}_1$
21: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff
$\vdash$ ($\uparrow$($\neg_b$(-a) $\leq$z -b)) $\in \mathbb{P}_1$
22: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff
$\vdash$ (-a) $\leq$z -b $\in \mathbb{B}$
23: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff
$\vdash$ (-a) $\in \mathbb{Z}$
24: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff
$\vdash$ (-b) $\in \mathbb{Z}$

25: . . . . .antecedent. . . . . NILNIL

3. $a \leq b$
4. $(\text{-}a) \leq \text{z}\ \text{-}b = \text{ff}$

$\vdash$ True

26: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. $(\text{-}a) \leq \text{z}\ \text{-}b = \text{ff}$
5. $(\uparrow(\neg_b(\text{-}a) \leq \text{z}\ \text{-}b)) = (\uparrow(\text{-}b) < \text{z}\ (\text{-}a))$

$\vdash \mathbb{P}_1 = \mathbb{P}_1$

27: . . . . .falsecase. . . . . NILNIL

3. $a \leq b$
4. $(\text{-}b) < (\text{-}a)$

$\vdash (\text{-}b) = (\text{-}b)$

28: . . . . .wf. . . . . NILNIL

3. $a \leq \text{z}\ b = \text{ff}$

$\vdash (a \leq \text{z}\ b = \text{ff}) \in \mathbb{P}_1$

29: . . . . .wf. . . . . NILNIL

3. $a \leq \text{z}\ b = \text{ff}$

$\vdash (\uparrow b < \text{z}\ a) \in \mathbb{P}_1$

30: . . . . .wf. . . . . NILNIL

3. $a \leq \text{z}\ b = \text{ff}$

$\vdash (b < a) \in \mathbb{P}_1$

31: . . . . .wf. . . . . NILNIL

3. $a \leq \text{z}\ b = \text{ff}$

$\vdash (\uparrow(\neg_b a \leq \text{z}\ b)) \in \mathbb{P}_1$

32: . . . . .wf. . . . . NILNIL

3. $a \leq \text{z}\ b = \text{ff}$

$\vdash a \leq \text{z}\ b \in \mathbb{B}$

33: . . . . .wf. . . . . NILNIL

3. $a \leq \text{z}\ b = \text{ff}$

$\vdash a \in \mathbb{Z}$

34: . . . . .wf. . . . . NILNIL

3. $a \leq \text{z}\ b = \text{ff}$

$\vdash b \in \mathbb{Z}$

35: . . . . .antecedent. . . . . NILNIL

3. $a \leq_{\mathrm{z}} b = \mathrm{ff}$
$\vdash$ True
36: . . . . .wf. . . . . NILNIL

3. $a \leq_{\mathrm{z}} b = \mathrm{ff}$
4. $(\uparrow(\neg_b a \leq_{\mathrm{z}} b)) = (\uparrow b <_{\mathrm{z}} a)$
$\vdash \mathbb{P}_1 = \mathbb{P}_1$
37: . . . . .wf. . . . . NILNIL

3. $b < a$
$\vdash (\text{-}a) \leq_{\mathrm{z}} \text{-}b \in \mathbb{B}$
38: . . . . .wf. . . . . NILNIL

3. $b < a$
$\vdash \mathbb{B} \in \mathrm{Type}$
39: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash ((\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}) \in \mathbb{P}_1$
40: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash (\uparrow(\text{-}a) \leq_{\mathrm{z}} \text{-}b) \in \mathbb{P}_1$
41: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash ((\text{-}a) \leq (\text{-}b)) \in \mathbb{P}_1$
42: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash (\text{-}a) \leq_{\mathrm{z}} \text{-}b \in \mathbb{B}$
43: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash (\text{-}a) \in \mathbb{Z}$
44: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq_{\mathrm{z}} \text{-}b = \mathrm{tt}$
$\vdash (\text{-}b) \in \mathbb{Z}$
45: . . . . .truecase. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq (\text{-}b)$
$\vdash (\text{-}a) = (\text{-}a)$
46: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq\text{z } \text{-}b = \text{ff}$
$\vdash ((\text{-}a) \leq\text{z } \text{-}b = \text{ff}) \in \mathbb{P}_1$
47: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq\text{z } \text{-}b = \text{ff}$
$\vdash (\uparrow(\text{-}b) <\text{z } (\text{-}a)) \in \mathbb{P}_1$
48: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq\text{z } \text{-}b = \text{ff}$
$\vdash ((\text{-}b) < (\text{-}a)) \in \mathbb{P}_1$
49: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq\text{z } \text{-}b = \text{ff}$
$\vdash (\uparrow(\neg_b(\text{-}a) \leq\text{z } \text{-}b)) \in \mathbb{P}_1$
50: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq\text{z } \text{-}b = \text{ff}$
$\vdash (\text{-}a) \leq\text{z } \text{-}b \in \mathbb{B}$
51: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq\text{z } \text{-}b = \text{ff}$
$\vdash (\text{-}a) \in \mathbb{Z}$
52: . . . . .wf. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq\text{z } \text{-}b = \text{ff}$
$\vdash (\text{-}b) \in \mathbb{Z}$
53: . . . . .antecedent. . . . . NILNIL

3. $b < a$
4. $(\text{-}a) \leq\text{z } \text{-}b = \text{ff}$
$\vdash$ True
54: . . . . .wf. . . . . NILNIL

3. $b < a$
4. (-a) $\leq$z -b = ff
5. ($\uparrow(\neg_b$(-a) $\leq$z -b)) = ($\uparrow$(-b) <z (-a))

$\vdash \mathbb{P}_1 = \mathbb{P}_1$

55: . . . . .falsecase. . . . . NILNIL

3. $b < a$
4. (-b) < (-a)

$\vdash$ (-a) = (-b)

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